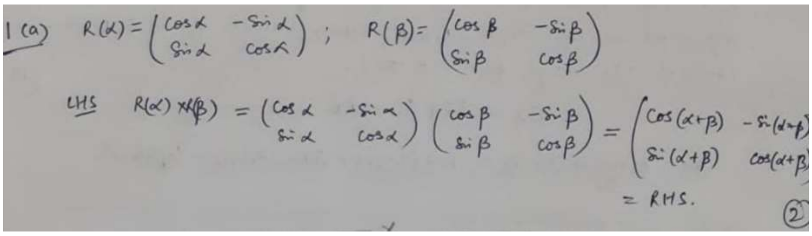
## Section A

Q1 a) Show that the composition of two rotations is additive. 2

R(α) x R(β) = R(α+β)



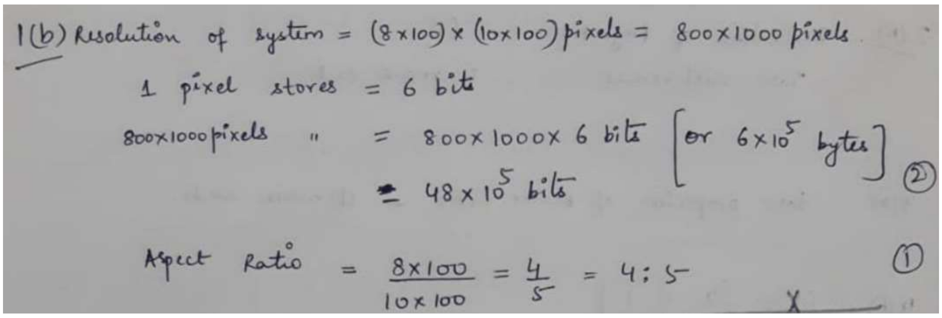
1 (a) $R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ ; $R(\beta) = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$

LHS $R(\alpha) \times R(\beta) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta \\ \sin(\alpha+\beta) & \cos(\alpha+\beta \end{pmatrix}$

$= RHS.$

②

b) Suppose an RGB raster system is to be designed using an 8 inch x 10 inch screen with a resolution of 100 pixels per inch in each direction. If we want to store 6 bits per pixel in the frame buffer, how much storage in bytes do we need for the frame buffer? Also find the aspect ratio of the raster system. 3 (2+1)



1(b) Resolution of system $= (8\times100)\times(10\times100)$ pixels $= 800\times1000$ pixels

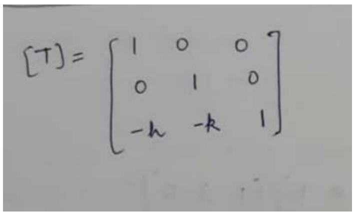
1 pixel stores $= 6$ bits

$800\times1000$ pixels " $= 800\times1000\times6$ bits [or $6\times10^5$ bytes] ②

$= 48\times10^5$ bits

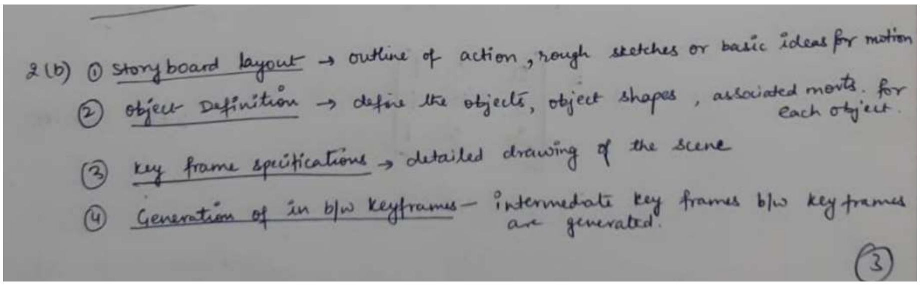
Aspect Ratio $= \frac{8\times100}{10\times100} = \frac{4}{5} = 4:5$ ①

Q2 a) Construct a translation matrix to translate a Point P from position (h, k) to the origin. 2



$[T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -h & -k & 1 \end{bmatrix}$

b) Discuss briefly the steps involved in design of animation sequence. 3



2 (b) ① Story board layout → outline of action, rough sketches or basic ideas for motion

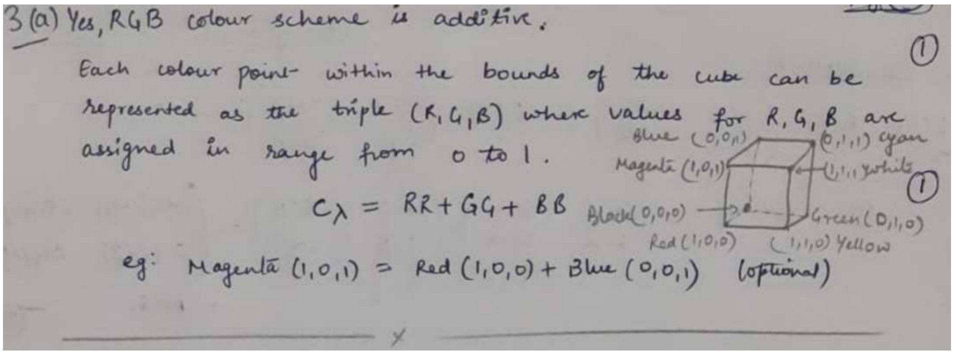
② object Definition → define the objects, object shapes, associated movts. for each object.

③ key frame specifications → detailed drawing of the scene

④ Generation of in b/w keyframes – intermediate key frames b/w key frames are generated.

③

Q3 a) Is RGB colour model additive? Justify your answer. 2



3 (a) Yes, RGB colour scheme is additive. ①

Each colour point within the bounds of the cube can be represented as the triple (R, G, B) where values for R, G, B are assigned in range from 0 to 1. ①

$$C_\lambda = RR + GG + BB$$

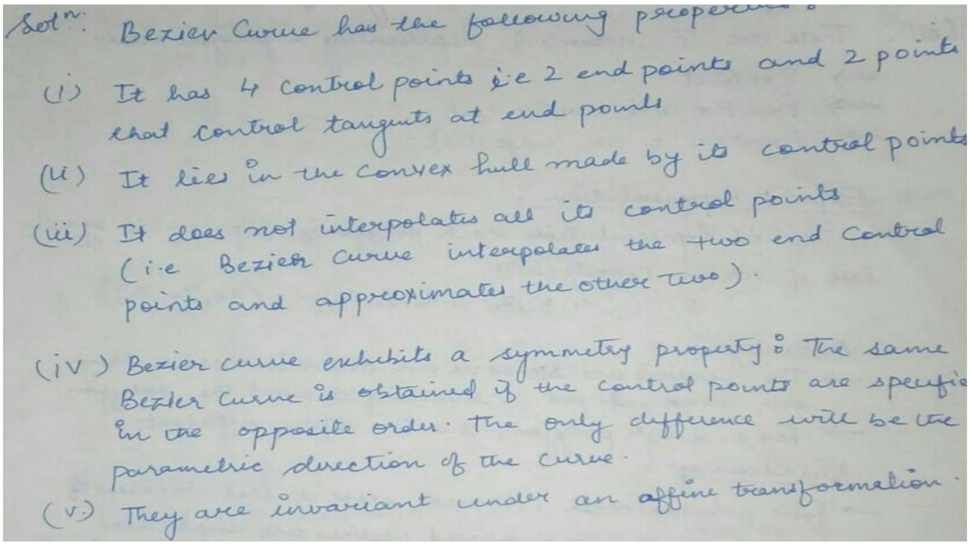
eg: Magenta (1,0,1) = Red (1,0,0) + Blue (0,0,1) (optional)

b) Define Projection. Give any two differences between parallel and perspective projections. 3 (1+2)

Projection of a 3D object is defined by straight projection rays (called projectors) emanating from a centre of projection (COP), passing through each point of object and intersecting a projection plane to form the projection.

| Perspective Projection | Parallel Projection |
|---|---|
| The distance from the center of projection to the projection plane is finite. | infinite |
| While defining it, we specify its COP. | We give its direction (DOP). |
| COP being a point has homogeneous co-ordinates of the form (x,y,z,1). | Since direction of projection is a vector (i.e. difference between points), it can be calculated by subtracting 2 points. i.e.<br>D= (x,y,z,1)-(x',y',z',1)=(a,b,c,0)<br>i.e., COP is at infinity. |
| It has visual effect similar to photographic system and human visual system and is known as perspective foreshortening. i.e. the size of the perspective projection of an object varies inversely with that of the distance of the object from COP. So, it has less realistic view. | Foreshortening is lacking. So it tends to look more real. |

Q4 a) Write any two properties of Bezier curve. 2



Sol$^n$: Bezier Curve has the following properties:

(i) It has 4 Control points i.e 2 end points and 2 points that control tangents at end points

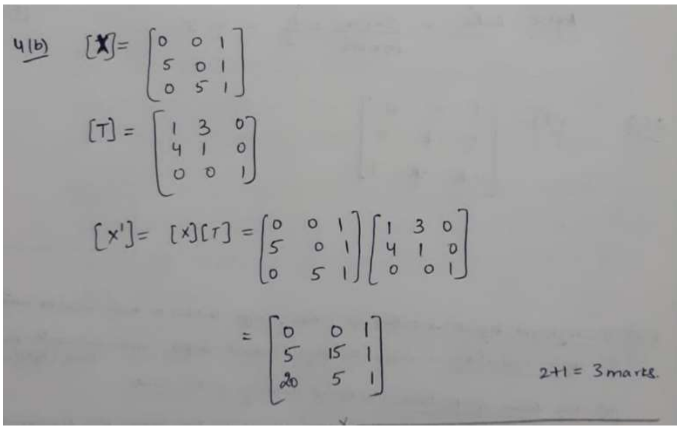
(ii) It lies in the convex hull made by its control points

(iii) It does not interpolates all its control points (i.e Bezier Curve interpolates the two end Control points and approximates the other two).

(iv) Bezier curve exhibits a symmetry property: The same Bezier Curve is obtained if the control points are specifie in the opposite order. The only difference will be the parametric direction of the curve.

(v) They are invariant under an affine transformation.

b) Consider a triangle ABC with A(0,0), B(5,0) and C(0,5). Give transformation matrix after shearing triangle ABC by 3 units along Y-axis and 4 units along X-axis. Use homogeneous coordinates. 3



4(b) $[X] = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 0 & 5 & 1 \end{bmatrix}$

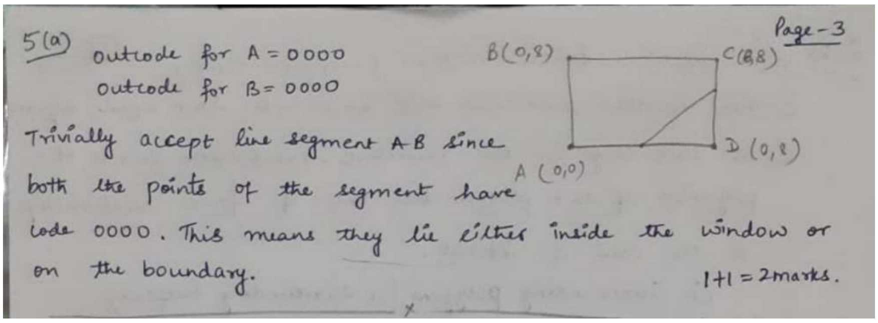
$[T] = \begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$[X'] = [X][T] = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 5 & 15 & 1 \\ 20 & 5 & 1 \end{bmatrix}$$

2+1 = 3 marks.

Q5 a) What is the condition for trivial acceptance of a line segment AB with A(0,4) and B(8,4) in Cohen Sutherland Line Clipping Algorithm using rectangular window coordinates as A(0,0), B(8,0), C(8,8) and D(0,8)? 2



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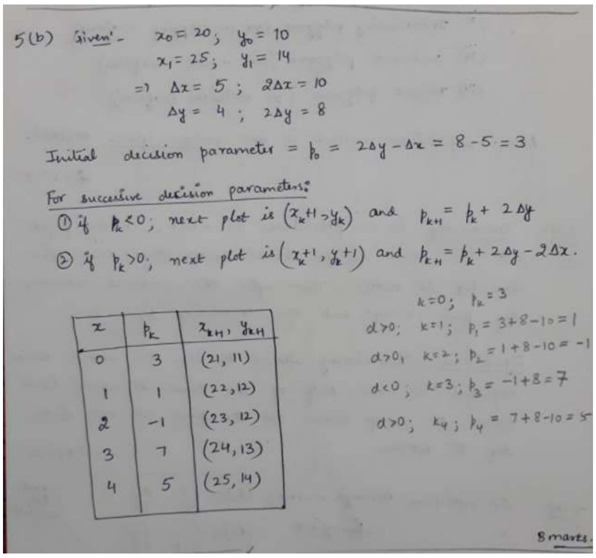
5(a) Outcode for A = 0000

Outcode for B = 0000

Trivially accept line segment AB since both the points of the segment have code 0000. This means they lie either inside the window or on the boundary.

1+1 = 2 marks.

b) Using Bresenham's line drawing algorithm find out the list of the rasterized pixels for the line from (20,10) to (25,14). 3



5(b) Given:- $x_0 = 20$; $y_0 = 10$

$x_1 = 25$; $y_1 = 14$

$\Rightarrow \Delta x = 5$; $2\Delta x = 10$

$\Delta y = 4$; $2\Delta y = 8$

Initial decision parameter $= p_0 = 2\Delta y - \Delta x = 8 - 5 = 3$

For successive decision parameters:

① if $p_k < 0$; next plot is $(x_k+1, y_k)$ and $p_{k+1} = p_k + 2\Delta y$

② if $p_k > 0$, next plot is $(x_k+1, y_k+1)$ and $p_{k+1} = p_k + 2\Delta y - 2\Delta x$.

| $x$ | $p_k$ | $x_{k+1}, y_{k+1}$ |
|---|---|---|
| 0 | 3 | (21, 11) |
| 1 | 1 | (22,12) |
| 2 | −1 | (23, 12) |
| 3 | 7 | (24,13) |
| 4 | 5 | (25, 14) |

$k=0$; $p_k = 3$

$d>0$; $k=1$; $p_1 = 3+8-10 = 1$

$d>0$, $k=2$; $p_2 = 1+8-10 = -1$

$d<0$, $k=3$; $p_3 = -1+8 = 7$

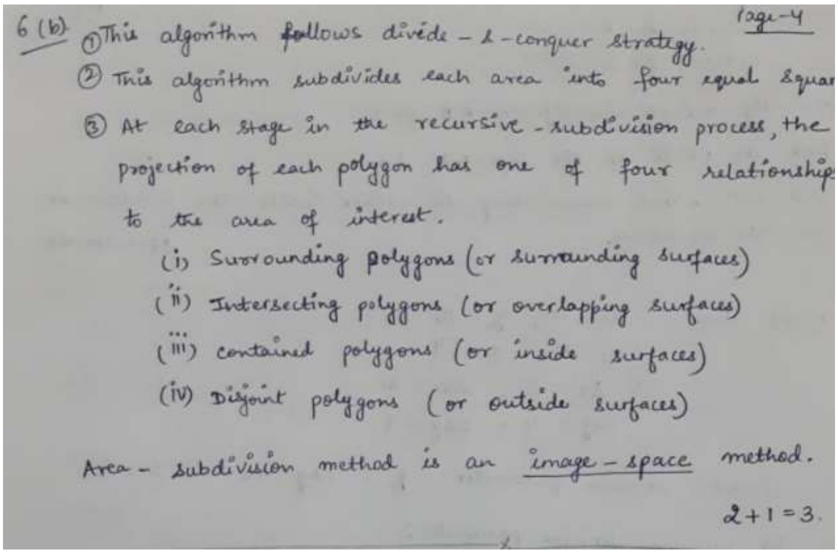
$d>0$; $k_4$; $p_4 = 7+8-10 = 5$

3 marks.

Q6 a) What is Specular reflection? 2

When we look at an illuminated shiny surface, such as polished metal, we see a highlight or bright spot, at certain viewing directions. This phenomenon is called specular reflection. Specular reflection is the result of total, or near total, reflection of the incident light in a concentrated region around the specular-reflection angle.

b) What are the steps in an Area-Subdivision method for Visible Surface determination? Is it an object-space method or image-space method? 3



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6 (b) ① This algorithm follows divide - & - conquer strategy.

② This algorithm subdivides each area into four equal squar

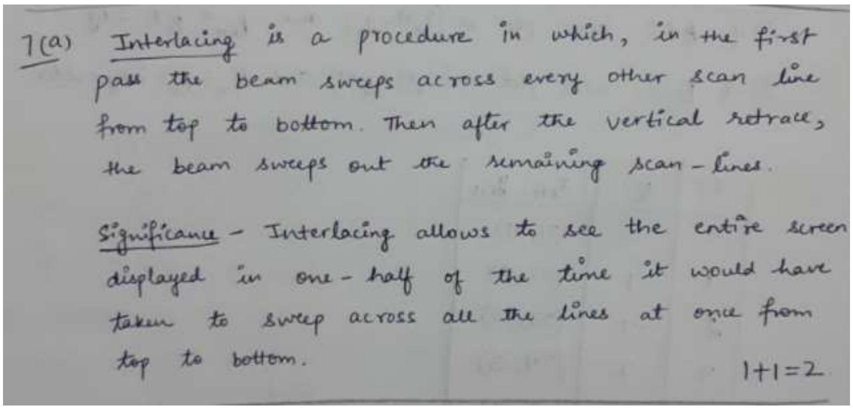
③ At each stage in the recursive - subdivision process, the projection of each polygon has one of four relationship to the area of interest.

(i) Surrounding polygons (or surrounding surfaces)
(ii) Intersecting polygons (or overlapping surfaces)
(iii) contained polygons (or inside surfaces)
(iv) Disjoint polygons (or outside surfaces)

Area - subdivision method is an image - space method.

2+1 = 3.

Q7 a) What is interlacing? Discuss its significance in raster graphics. 2

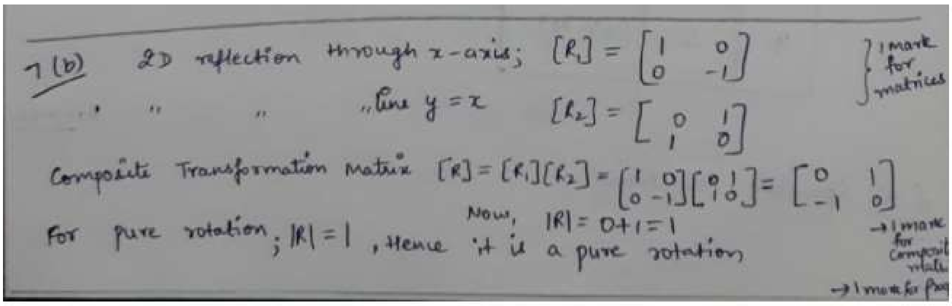


7(a) Interlacing is a procedure in which, in the first pass the beam sweeps across every other scan line from top to bottom. Then after the vertical retrace, the beam sweeps out the remaining scan - lines.

Significance - Interlacing allows to see the entire screen displayed in one - half of the time it would have taken to sweep across all the lines at once from top to bottom.

1+1=2.

b) Show that a 2D reflection through the x-axis, followed by a 2D reflection through the line y=x, is equivalent to a pure rotation about the origin. 3



7 (b) 2D reflection through x-axis; $[R_1] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

" " " , line y = x $[R_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

} 1 mark for matrices

Composite Transformation Matrix $[R] = [R_1][R_2] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Now, $|R| = 0+1 = 1$

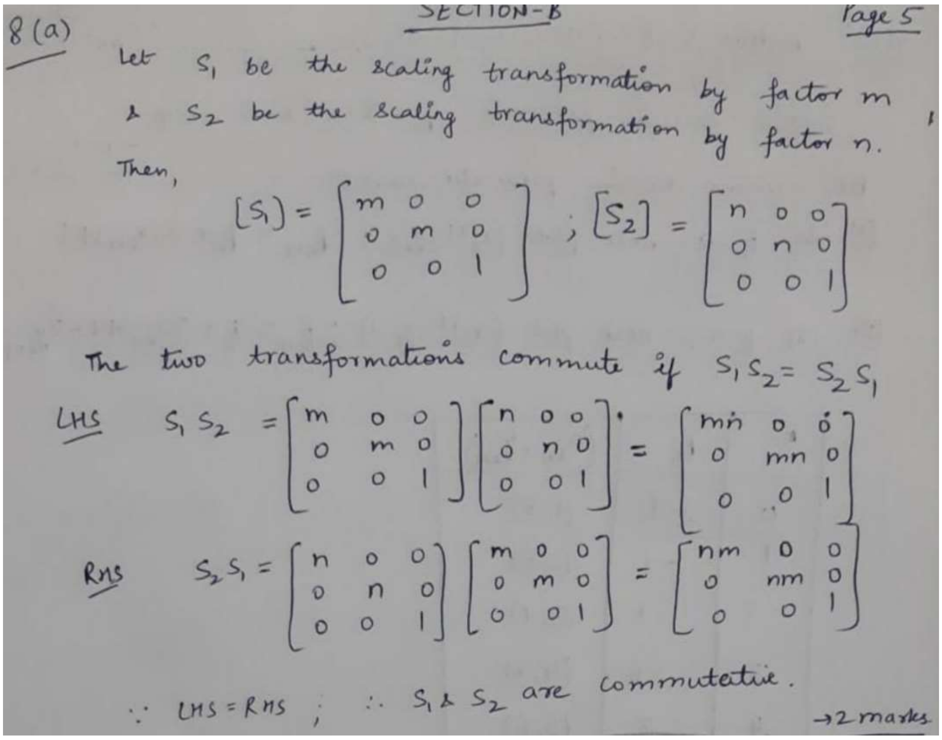
For pure rotation; $|R| = 1$, Hence it is a pure rotation

→ 1 mark for Composit

→ 1 mark for

## Section B

Q8 a) Prove that two scaling transformations are commutative. 4 (2+1+1)



8 (a) SECTION-B Page 5

Let $S_1$ be the scaling transformation by factor m & $S_2$ be the scaling transformation by factor n.

Then,

$$[S_1] = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} ; [S_2] = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The two transformations commute if $S_1 S_2 = S_2 S_1$

LHS $$S_1 S_2 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} mn & 0 & 0 \\ 0 & mn & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

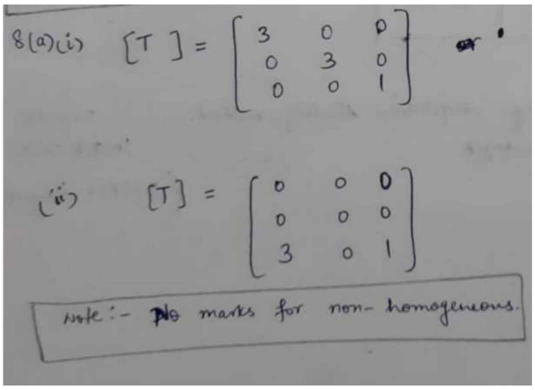
RHS $$S_2 S_1 = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} nm & 0 & 0 \\ 0 & nm & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∵ LHS = RHS ; ∴ $S_1$ & $S_2$ are commutative.

→2 marks

Write 3X3 2-D transformation matrix for each of the following transformations respectively:

- (i) Enlarge the object by three times.
- (ii) Translate the object by 3 units in x direction.

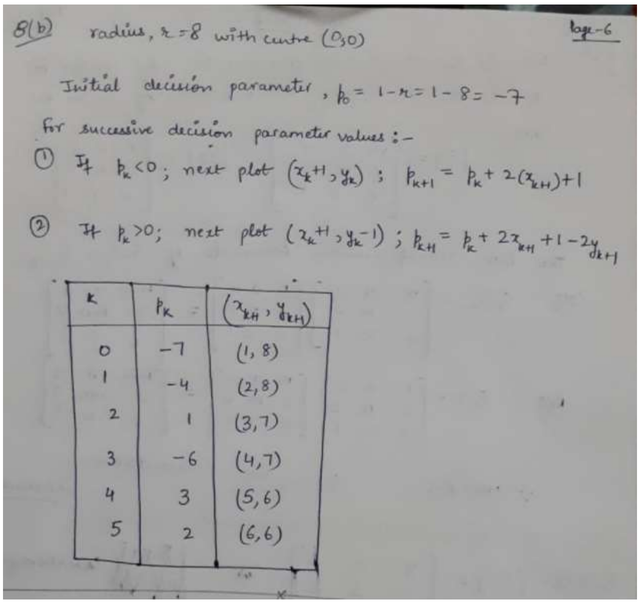


8(a)(i) $$[T] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ or

(ii) $$[T] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Note :- No marks for non-homogeneous.

b) Using mid-point circle drawing algorithm find out the pixel positions lying in the first quadrant of the circle with centre at (0,0) and radius of 8 units. 6



8(b) radius, $r = 8$ with centre (0,0)　　Page-6

Initial decision parameter, $p_0 = 1 - r = 1 - 8 = -7$

for successive decision parameter values :-

① If $p_k < 0$; next plot $(x_k+1, y_k)$ ; $p_{k+1} = p_k + 2(x_{k+1}) + 1$

② If $p_k > 0$; next plot $(x_k+1, y_k-1)$ ; $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

| k | $p_k$ | $(x_{k+1}, y_{k+1})$ |
|---|---|---|
| 0 | -7 | (1, 8) |
| 1 | -4 | (2,8) |
| 2 | 1 | (3,7) |
| 3 | -6 | (4,7) |
| 4 | 3 | (5,6) |
| 5 | 2 | (6,6) |

Q9 a) Describe Phong interpolation shading method for polygon rendering. Give any two advantages of this method. 4 (2+2)

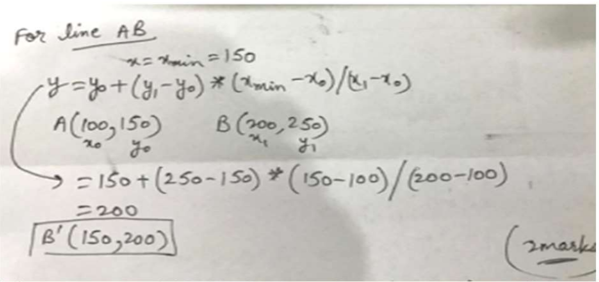
Steps:

1. Determine the average unit normal vector at each vertex
2. Linearly interpolate the vertex normal over the projected area of the polygon
3. Apply an illumination model at positions along scan lines to calculate pixel intensities

Advantages:

- Gouraud shading is faster than Phong shading
- Phong shading is more accurate. Intensity calculations using an approximated normal vector at each point along the scan line produce more accurate results than the direct interpolation of intensities, as in Gouraud shading.

b) Using Sutherland Hodgman Polygon Clipping Algorithm, clip the polygon ABC with coordinates A(100,150), B(200,250) and C(300,200) against the clipping window with coordinates P(150,150), Q(150,200), R(200,200) and S(200,150). 6



For line AB

$x = x_{min} = 150$

$y = y_0 + (y_1 - y_0) * (x_{min} - x_0)/(x_1 - x_0)$

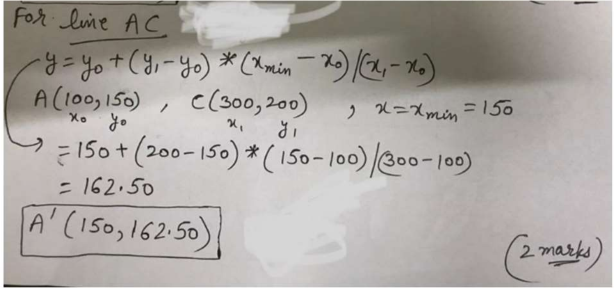
$A(\underset{x_0}{100}, \underset{y_0}{150})$　　$B(\underset{x_1}{200}, \underset{y_1}{250})$

$= 150 + (250 - 150) * (150 - 100)/(200 - 100)$

$= 200$

$\boxed{B'(150,200)}$

(2marks



For line AC

$y = y_0 + (y_1 - y_0) * (x_{min} - x_0)/(x_1 - x_0)$

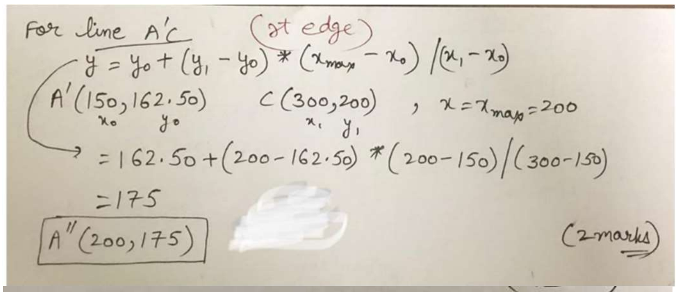
$A(\underset{x_0}{100}, \underset{y_0}{150})$ , $C(\underset{x_1}{300}, \underset{y_1}{200})$ , $x = x_{min} = 150$

$= 150 + (200 - 150) * (150 - 100)/(300 - 100)$

$= 162.50$

$\boxed{A'(150, 162.50)}$

(2 marks)



For line A'C (3rd edge)

$y = y_0 + (y_1 - y_0) * (x_{max} - x_0) / (x_1 - x_0)$

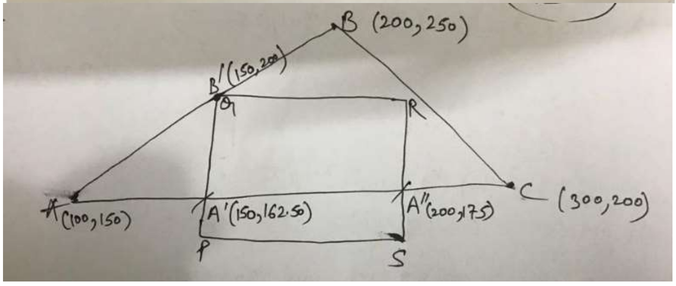
$A'(150, 162.50)$ with $x_0$, $y_0$ below; $C(300, 200)$ with $x_1$, $y_1$ below; $x = x_{max} = 200$

$= 162.50 + (200 - 162.50) * (200 - 150) / (300 - 150)$

$= 175$

$\boxed{A''(200, 175)}$

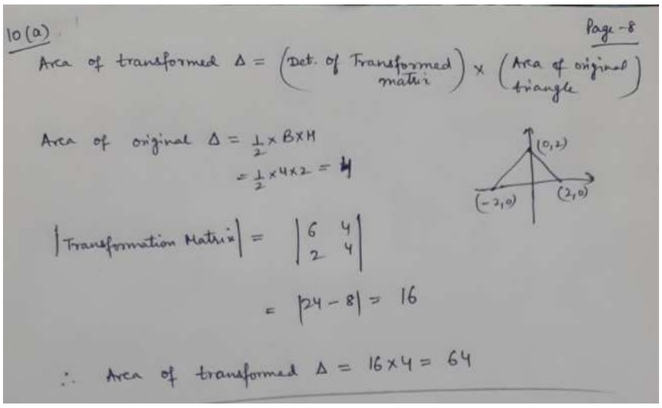
(2 marks)



Q10 a) A triangle is defined by vertices (2,0), (0,2), (-2,0). It is transformed by 2x2 transformation matrix 4

$$T = \begin{bmatrix} 6 & 4 \\ 2 & 4 \end{bmatrix}$$

Find the area of transformed triangle.



10(a) Page -8

Area of transformed Δ = (Det. of Transformed matrix) × (Area of original triangle)

Area of original Δ = $\frac{1}{2} \times B \times H$

$= \frac{1}{2} \times 4 \times 2 = 4$

$|\text{Transformation Matrix}| = \begin{vmatrix} 6 & 4 \\ 2 & 4 \end{vmatrix}$

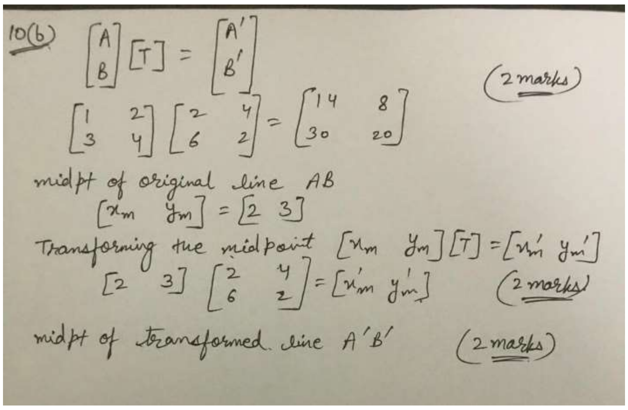
$= |24 - 8| = 16$

∴ Area of transformed Δ = $16 \times 4 = 64$

b) Consider a line AB with position vectors of end point as [A] = [ 1 2] and [B] = [ 3 4 ]. 6

The transformation matrix is given as $[T] = \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix}$

Calculate the transformed line A'B'. Also prove that the midpoint of original line AB yield same results for the midpoint of transformed line A'B'.



10(b) $\begin{bmatrix} A \\ B \end{bmatrix} [T] = \begin{bmatrix} A' \\ B' \end{bmatrix}$ (2 marks)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 30 & 20 \end{bmatrix}$

midpt of original line AB

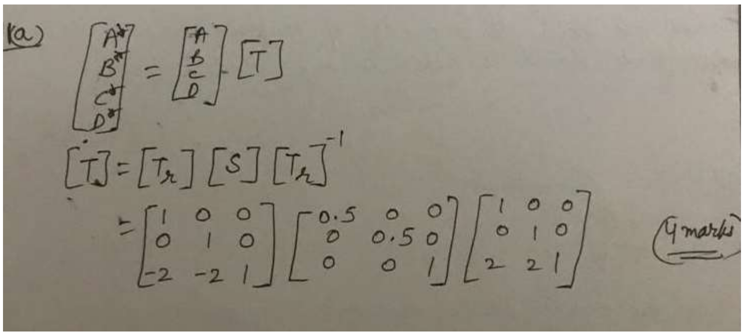
$[x_m \quad y_m] = [2 \quad 3]$

Transforming the midpoint $[x_m \quad y_m][T] = [x_m' \quad y_m']$

$[2 \quad 3] \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix} = [x_m' \quad y_m']$ (2 marks)

midpt of transformed line A'B' (2 marks)

Q11 a) Consider a square ABCD with coordinates as A(0,0), B(0,4), C(4,4) and D(4,0). Let the centre of the square be at coordinate P(2,2). Apply 2-D transformation to reduce the square ABCD to half of its size, with centre fixed at point P. 4 (1+2+1)



(a)

$$\begin{bmatrix} A^* \\ B^* \\ C^* \\ D^* \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} [T]$$

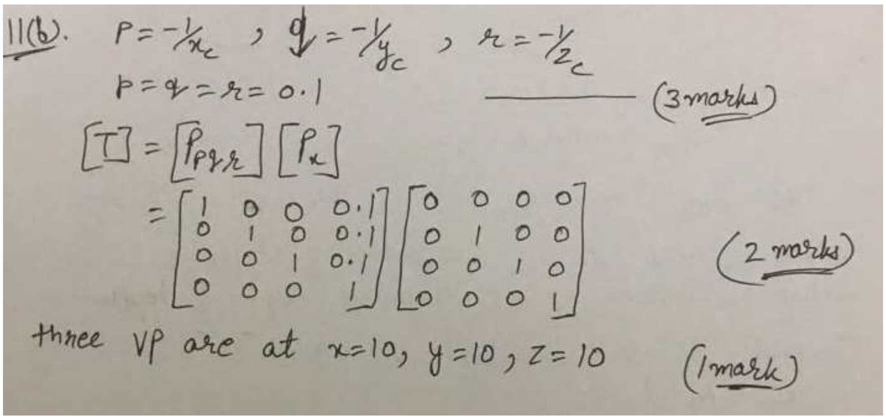
$$[T] = [T_r][S][T_r]^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

(4 marks)

b) Perform a 3-point perspective projection onto the x=0 plane on a unit cube with centre of projections at $x_c$= -10, $y_c$ = -10 and $z_c$ = -10. Also, give the vanishing points. Consider the coordinates of the unit cube as follows: 6 (3+2+1)

$$[X]=\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



11(b). $P = -1/x_c$ , $q = -1/y_c$ , $r = -1/z_c$

$p = q = r = 0.1$ ———— (3 marks)

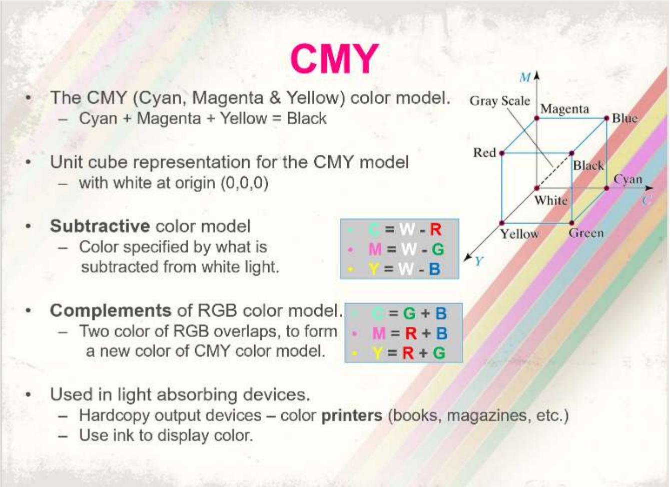
$$[T] = [P_{pqr}][P_x]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0.1 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2 marks)

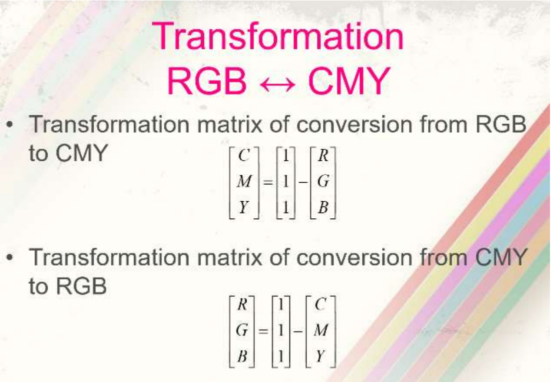
three VP are at $x=10$, $y=10$, $z=10$ (1 mark)

Q12 a) Explain CMY color model in graphics system. 4



## CMY

- The CMY (Cyan, Magenta & Yellow) color model.
  - Cyan + Magenta + Yellow = Black
- Unit cube representation for the CMY model
  - with white at origin (0,0,0)
- **Subtractive** color model
  - Color specified by what is subtracted from white light.
  - C = W - R
  - M = W - G
  - Y = W - B
- **Complements** of RGB color model.
  - Two color of RGB overlaps, to form a new color of CMY color model.
  - C = G + B
  - M = R + B
  - Y = R + G
- Used in light absorbing devices.
  - Hardcopy output devices – color **printers** (books, magazines, etc.)
  - Use ink to display color.



## Transformation RGB ↔ CMY

- Transformation matrix of conversion from RGB to CMY

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Transformation matrix of conversion from CMY to RGB

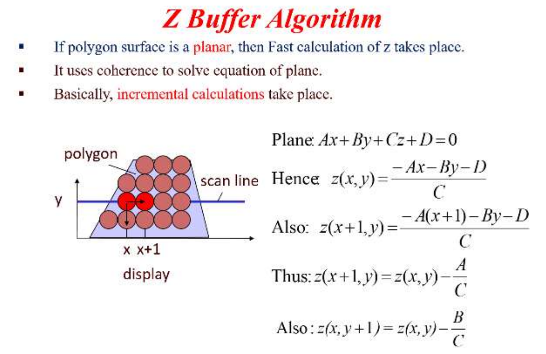
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

b) What do you mean by hidden surfaces? Explain Z-Buffer algorithm for visible surface determination. 6

- There are two buffers:
  - Frame Buffer: color values are stored for each pixel.
  - Z Buffer: depth values are stored for each pixel.
- There are two planes:
  - Back Clipping Plane
  - Front Clipping Plane

- The Z buffer is initialized to zero, representing the z value at the back clipping plane. The frame buffer is initialized to the background color. Polygons are scan-converted in the frame buffer in an arbitrary order. User eye is at positive infinity of Z axis.

If the polygon being scan-converted at a pixel is no farther from the viewer than is the point whose color and depth are currently in the buffers, then the new point's color and depth replace the old values.



## *Z Buffer Algorithm*

- If polygon surface is a planar, then Fast calculation of z takes place.
- It uses coherence to solve equation of plane.
- Basically, incremental calculations take place.

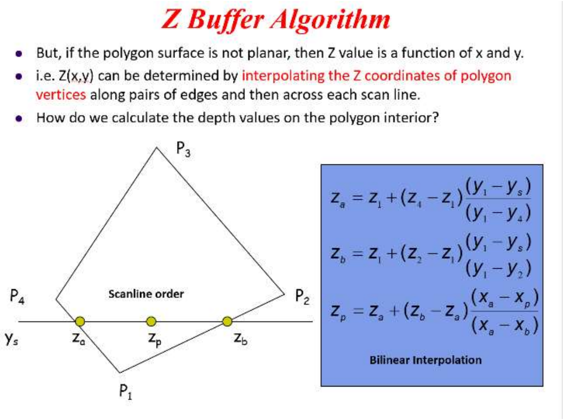
Plane: $Ax+By+Cz+D=0$

Hence: $z(x,y)=\dfrac{-Ax-By-D}{C}$

Also: $z(x+1,y)=\dfrac{-A(x+1)-By-D}{C}$

Thus: $z(x+1,y)=z(x,y)-\dfrac{A}{C}$

Also : $z(x,y+1)=z(x,y)-\dfrac{B}{C}$



## *Z Buffer Algorithm*

- But, if the polygon surface is not planar, then Z value is a function of x and y.
- i.e. Z(x,y) can be determined by interpolating the Z coordinates of polygon vertices along pairs of edges and then across each scan line.
- How do we calculate the depth values on the polygon interior?

$$z_a = z_1 + (z_4 - z_1)\frac{(y_1 - y_s)}{(y_1 - y_4)}$$

$$z_b = z_1 + (z_2 - z_1)\frac{(y_1 - y_s)}{(y_1 - y_2)}$$

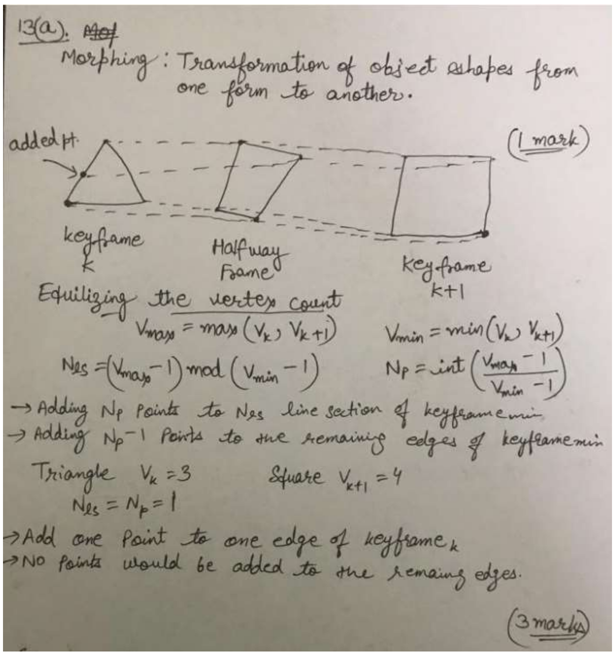
$$z_p = z_a + (z_b - z_a)\frac{(x_a - x_p)}{(x_a - x_b)}$$

**Bilinear Interpolation**

Advantages: simple and fast

Disadvantages: costs memory, transparency is tricky.

Q13 a) What is Morphing? Morph a triangle into a square by equalizing the vertex count. 4



13(a). ~~Mof~~

Morphing : Transformation of object shapes from one form to another.

(1 mark)

Equilizing the vertex count

$V_{max} = max(V_k, V_{k+1})$ $V_{min} = min(V_k, V_{k+1})$

$N_{ls} = (V_{max} - 1) \bmod (V_{min} - 1)$ $N_p = int\left(\frac{V_{max} - 1}{V_{min} - 1}\right)$

→ Adding $N_p$ Points to $N_{ls}$ line section of $keyframe_{min}$

→ Adding $N_p - 1$ Points to the remaining edges of $keyframe_{min}$

Triangle $V_k = 3$ Square $V_{k+1} = 4$

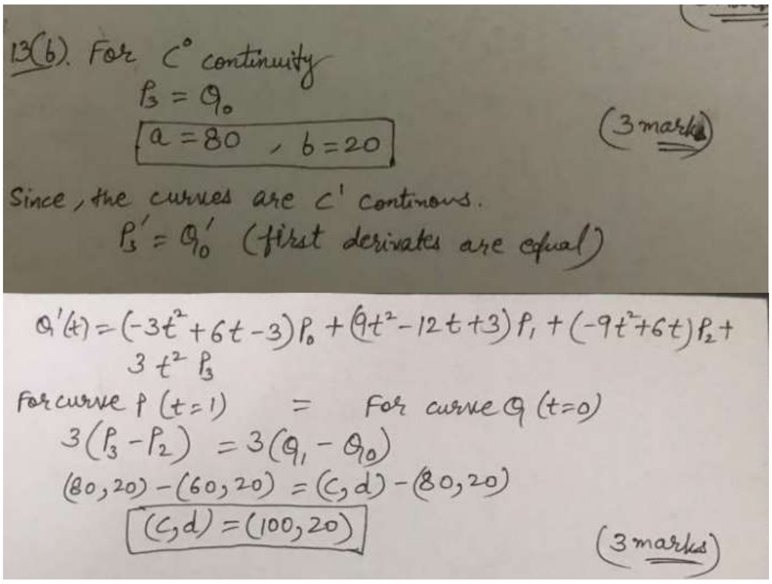
$N_{ls} = N_p = 1$

→ Add one Point to one edge of $keyframe_k$

→ No Points would be added to the remaing edges.

(3 marks)

b) Consider two Bezier curve segments defined by control points $P_0(20,20)$, $P_1(40,50)$, $P_2(60,20)$ and $P_3(80,20)$. Another curve segment is defined by $Q_0(a,b)$, $Q_1(c,d)$, $Q_2$ and $Q_3$. Find the point $Q_0$ and $Q_1$ such that two curve join smoothly and $C^1$ continuity exists between them. 6



13(b). For $C^0$ continuity

$P_3 = Q_0$

$\boxed{a = 80, \ b = 20}$

(3 marks)

Since, the curves are $C^1$ continous.

$P_3' = Q_0'$ (first derivates are equal)

$$Q'(t) = (-3t^2 + 6t - 3)P_0 + (9t^2 - 12t + 3)P_1 + (-9t^2 + 6t)P_2 + 3t^2 P_3$$

For curve P (t=1) = For curve Q (t=0)

$$3(P_3 - P_2) = 3(Q_1 - Q_0)$$

$$(80,20) - (60,20) = (c,d) - (80,20)$$

$$\boxed{(c,d) = (100,20)}$$

(3 marks)

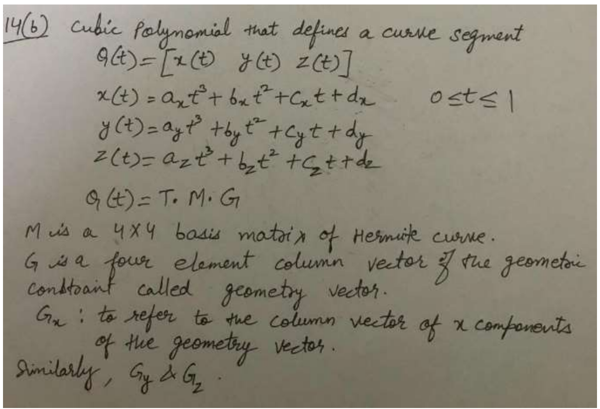
Q14 a) What is dithering? What are its advantages over halftoning? 4 (3+1)

- The term dithering refers to techniques for approximating halftones without reducing resolution.
- In Dithering, random values added to pixel intensities to break up contours are often referred to as dither noise.
- The effect is to add noise over an entire picture, which tends to soften intensity boundaries.
- To obtain $n^2$ intensity levels, we set up an $n$ by **n** dither matrix D, whose elements are distinct positive integers in the range **0** to $n^2 - 1$.
- Another method for mapping a picture with ***m*** by ***n*** points to a display area with m by n pixels is ***error diffusion.***

Advantages:

Dithering is a technique used in computer graphics to create the illusion of color depth in images with a limited color palette (color quantization). In a dithered image, colors not available in the palette are approximated by a diffusion of colored pixels from within the available palette. The human eye perceives the diffusion as a mixture of the colors within it (see color vision).

b) Derive the basis matrix for Hermite curve. 6



14(b) Cubic Polynomial that defines a curve segment

$$Q(t) = [x(t) \quad y(t) \quad z(t)]$$

$$x(t) = a_x t^3 + b_x t^2 + c_x t + d_x \qquad 0 \le t \le 1$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t + d_y$$

$$z(t) = a_z t^3 + b_z t^2 + c_z t + d_z$$

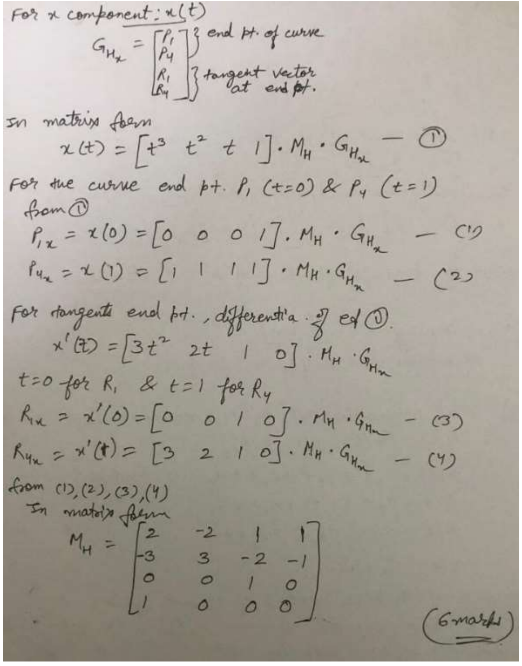
$$Q(t) = T \cdot M \cdot G$$

M is a 4X4 basis matrix of Hermite curve.

G is a four element column vector of the geometric constraint called geometry vector.

$G_x$ : to refer to the column vector of x components of the geometry vector.

Similarly, $G_y$ & $G_z$.



For x component: x(t)

$$G_{H_x} = \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix} \begin{matrix} \} \text{ end pt. of curve} \\ \\ \} \text{ tangent vector at end pt.} \end{matrix}$$

In matrix form

$$x(t) = [t^3 \quad t^2 \quad t \quad 1] \cdot M_H \cdot G_{H_x} \quad — ①$$

For the curve end pt. $P_1$ $(t=0)$ & $P_4$ $(t=1)$

from ①

$$P_{1x} = x(0) = [0 \quad 0 \quad 0 \quad 1] \cdot M_H \cdot G_{H_x} \quad — (1)$$

$$P_{4_x} = x(1) = [1 \quad 1 \quad 1 \quad 1] \cdot M_H \cdot G_{H_x} \quad — (2)$$

For tangents end pt., differentia. of eq ①.

$$x'(t) = [3t^2 \quad 2t \quad 1 \quad 0] \cdot M_H \cdot G_{H_x}$$

$t=0$ for $R_1$ & $t=1$ for $R_4$

$$R_{1x} = x'(0) = [0 \quad 0 \quad 1 \quad 0] \cdot M_H \cdot G_{H_x} \quad - (3)$$

$$R_{4x} = x'(t) = [3 \quad 2 \quad 1 \quad 0] \cdot M_H \cdot G_{H_x} \quad - (4)$$

from (1), (2), (3), (4)

In matrix form

$$M_H = \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(6 marks)